Solving Helmholtz Equation using Deep Neural Networks

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USC Viterbi School of Engineering

SHINE Summer High School Intensive in Next-Generation Engineering

Objective & Impact of Professor's Research

Professor Assad Oberai's research on neural networks covers a wide range of applications including solving inverse problems in physics, image processing for medical diagnosis, and exploring physics informed neural networks (PINNS). PINNS is a specific type of neural network in which a computer is "trained to solve supervised learning tasks while respecting any given laws of physics." As a relatively new method of solving complex differential equations, there are still much to discover in PINNS and it's unique potential. This research involving Ph.D students Harisankar Ramasamy and Orazio Pinti specifically focused on PINNS's ability to solve the Helmholtz Equation given differing values of wavenumber (k) and the implementation of the Fourier method.

Methods

Helmholtz Equation: commonly used to model the propagation of acoustic, electromagnetic, and quantum waves

ex) modeling the vibration of a taut string

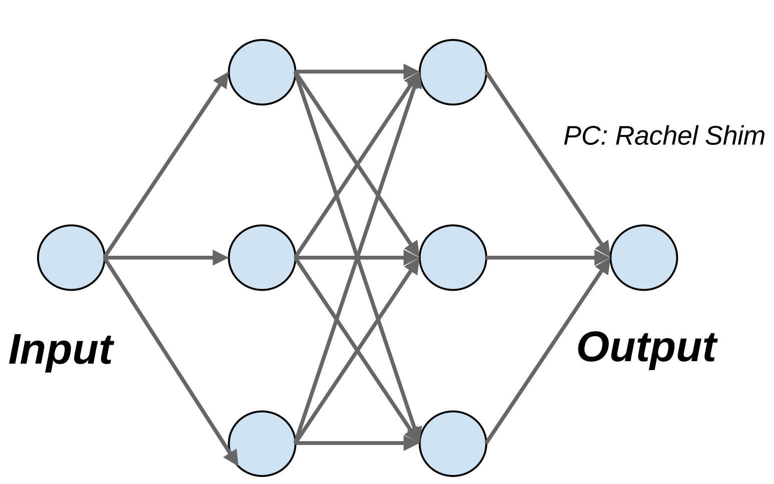
$$-\frac{d^2u}{dx^2} - k^2u = 0, \; x \in [0,1]$$

$$u(0) = 0$$

$$u(1) = 1$$

***Helmholtz Equation**; k (wavenumber)*

Neural Network: type of machine learning in which a computer learns to perform a certain task by analyzing training data



Neural Network

Loss Function:

$$L_{total} = L_{PDE} + \lambda_b L_{boundary}$$

Parameters	
- Width: 30	- Alpha: 1e-3
- Depth: 4	- Activation function: sin(x)
- Optimizer: Adam	

Loss vs k and λ_b

- K {3.5π, 4.5π, … 9.5π, 10.5π}
- λ_b {10, 100, 1k, 10k}
- Increase epochs until convergence
- N: 50

'N' is the number of prediction points used to compute the loss

Objective: Observe effects of increasing k & λ_b

Loss & Avg Error vs k & n with Fourier Features Input Mapping

- K {3.5π, 4.5π, … 9.5π, 10.5π}
- N {20, 40, 60, 80, 100}
- Epochs: 3000
- λ_b: 1000

Fourier Method: method of propagation that decomposes functions into sinusoidal components

Objective: Observe effects of Fourier method & increasing N

Results

Log(Total Loss) vs. Epochs *PC: Rachel Shim*

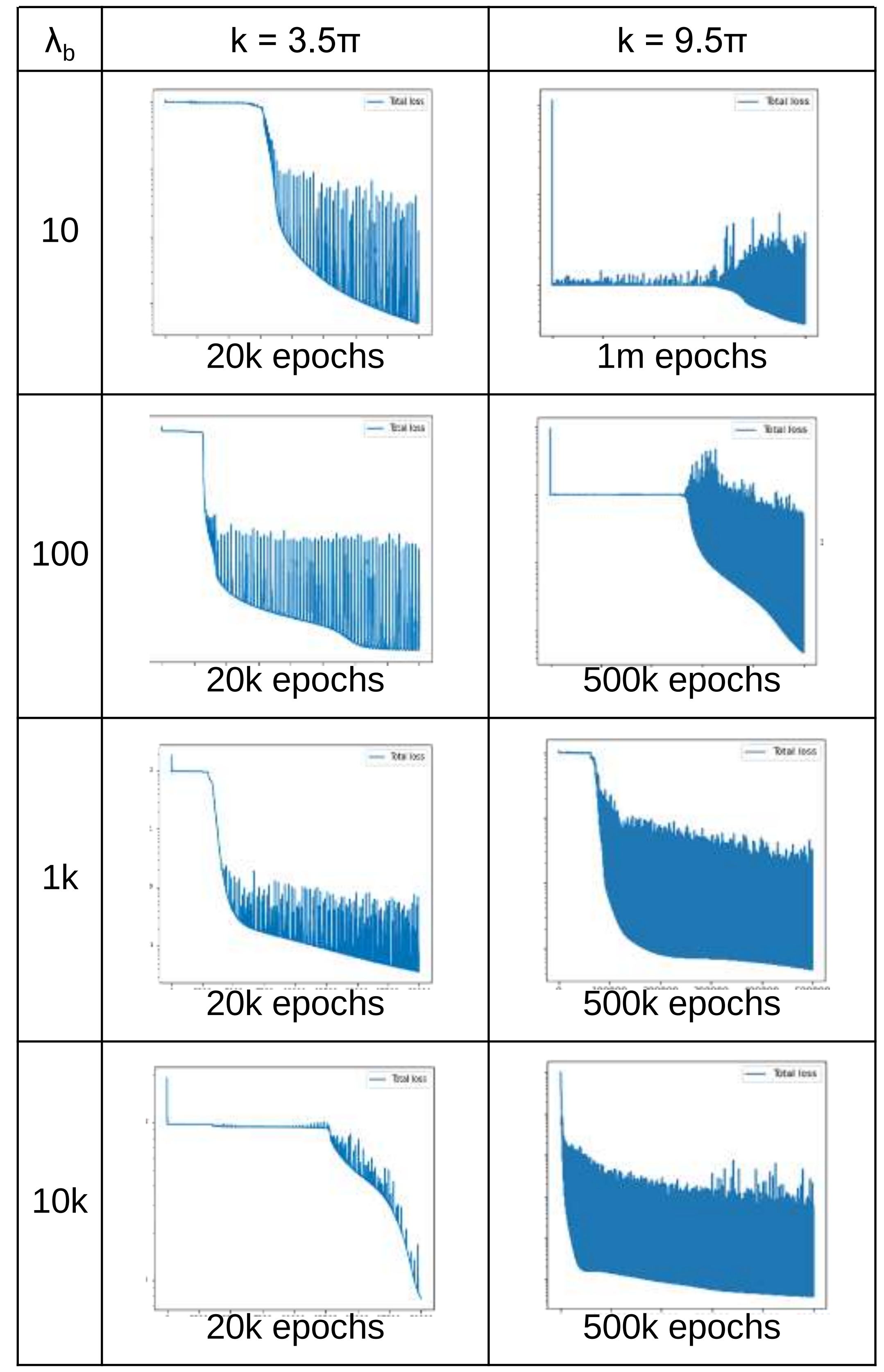


Figure 1

Figure 1:
- Greater k value requires greater # of epochs for convergence
- λ_b value becomes more significant with greater k value (requires greater λ_b for convergence)

Figure 2:
- Fourier's method significantly reduces # of epochs required
- Convergence occurs much faster (~150k vs <3k)

Figure 3:
- Greater N value lessens average error for greater k values
- For k values less than 10.5π, the average error remains constant given values of N

Comparing Standard & Fourier's Feature PINNs

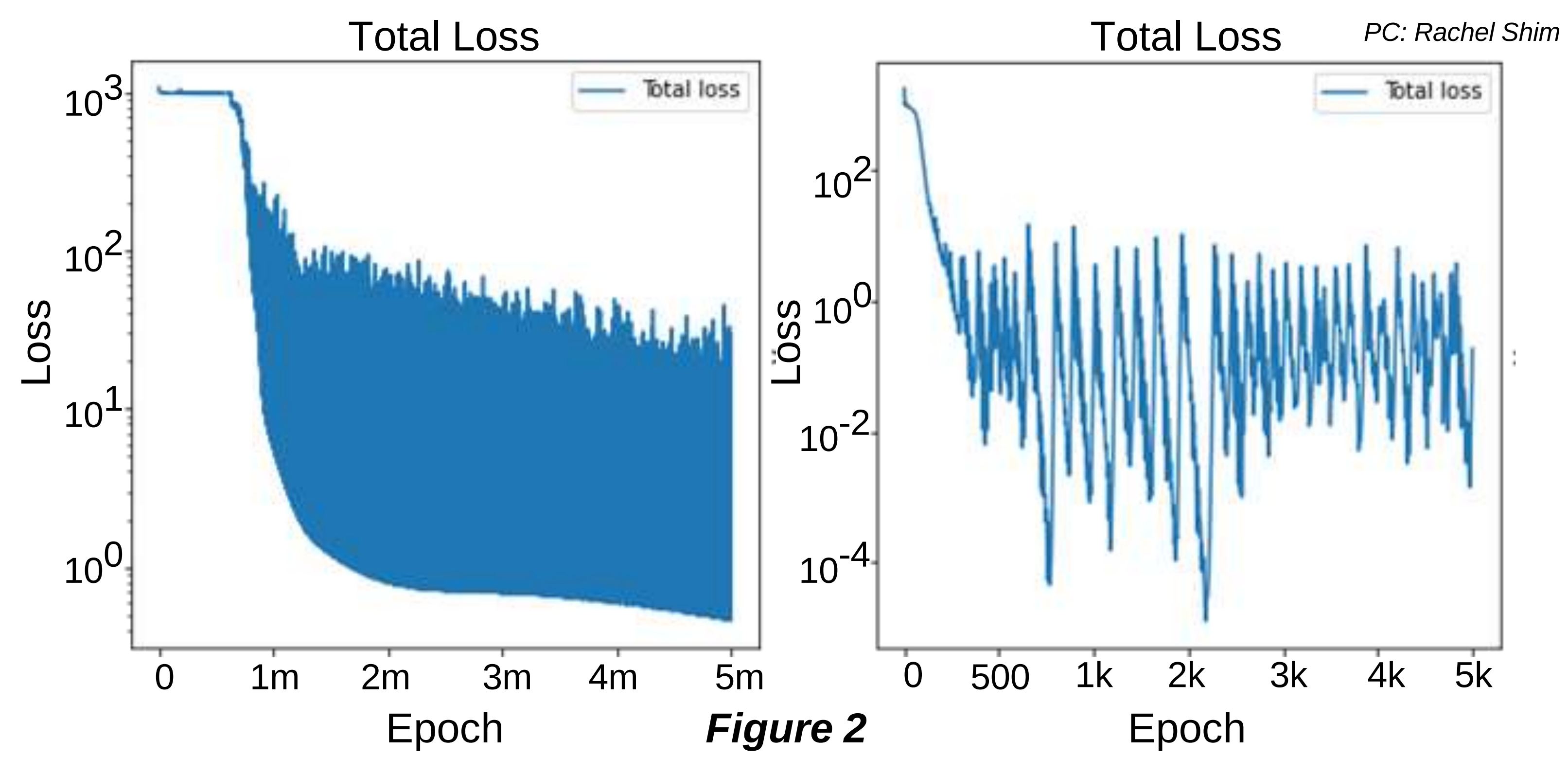


Figure 2

Avg Error vs. K *PC: Rachel Shim*

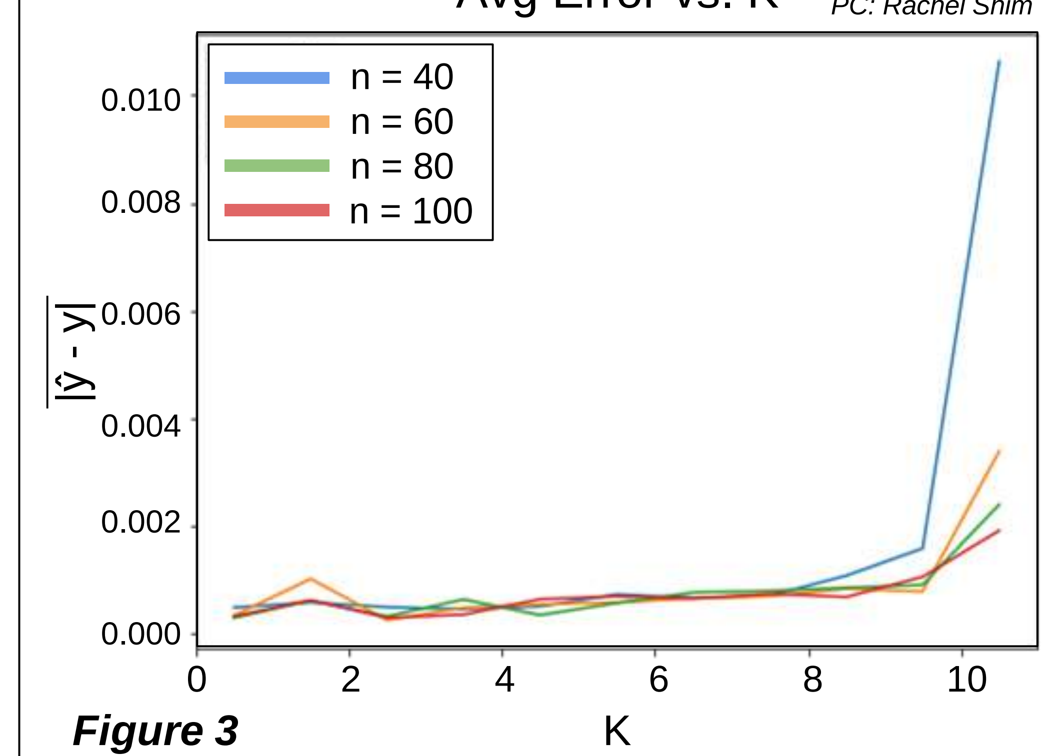


Figure 3

Applications to STEM Coursework/Next Steps

While conducting research on the Helmholtz equation, I came across familiar topics from my calculus, statistics, and physics classes. It was interesting to see how materials from three different courses combined to create the framework for PINNS and its innerworkings. I also learned new skills and concepts such as PDEs, gradient descent, tensorflow, numpy, and python plotting. PINNS is still a relatively new type of machine learning, and its potential is not yet fully discovered. Thus, as part of my engineering journey, I would like to further explore neural networks in the future as well as continue developing new skills in coding and mathematics.

Acknowledgements

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